

Question Paper Code: 41310

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018 Third Semester Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS (Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Computer Science and Engineering/Computer and Communication Engineering/ Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/ Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/ Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Information Technology/ Petrochemical Technology/Petroleum Engineering/Plastic Technology/ Polymer Technology)

(Regulations 2013)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions.

PART - A

 $(10\times2=20 \text{ Marks})$

1. Find the complete integral of the PDE: $z = px + qy + \sqrt{pq}$

2. Solve: $(D^3 - 3DD'^2 + 2D'^3) z = 0$

3. Find b_n in the expansion of $f(x) = x^2$ as a Fourier series in $(-\pi, \pi)$.

4. Define Root mean square value of a function.

5. Classify the partial differential equation $u_{xy} = u_x u_y + xy$.

6. State possible solutions of the one dimensional heat equation.

7. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{in } |x| < a \\ 0 & \text{in } |x| > a \end{cases}$

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- 8. State the convolution theorem for Fourier transforms.
- 9. Find the Z-transform of {n}.
- 10. Prove that $Z\{nf(n)\} = -z \frac{d}{dz} F(z)$, where $Z\{f(n)\} = F(z)$.

(5×16=80 Marks)

PART - B

(8)

- 11. a) i) Find the singular solution of the equation $z = px + qy + p^2 + pq + q^2$. (8)
 - ii) Solve: x(y z) p + y(z x) q = z(x y).

- (8) b) i) Solve: $(D^2 + 4DD' - 5D'^2)z = \sin(x - 2y) + e^{2x - y}$.
 - ii) Solve: $(D^2 + DD' 6D'^2)z = v\cos x$.

(8)

(8)

12. a) i) Find the Fourier series for $f(x) = x^2$ in $-\pi < x < \pi$.

- ii) Find the half range cosine series for $f(x) = x (\pi x)$ in $(0, \pi)$.

(8)

(16)

(OR)

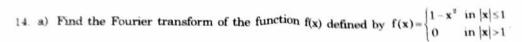
b) Find the Fourier series expansion upto the first three harmonics for the (16)function defined in the following table

| х | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | π | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | 2 π |
|---|---|-----------------|------------------|-----|------------------|------------------|-----|
| у | 1 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

13. a) A string is stretched and fastened to two points x = 0 and x = l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time t = 0. Find the displacement of any point on the string at a distance of x from one end at time t. (16)

(OR)

b) A bar, 10 cm long with insulated sides, has its ends A and B kept at $20^{\circ}\mathrm{C}$ and $40^{\circ}\mathrm{C}\$ respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50°C and at the same instant that at B is lowered to 10°C. Find the subsequent temperature at any point of the bar at any time.



Hence prove that $\int_0^\infty \frac{\sin s - s \cos s}{s^3} \cos \left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$. Also show that

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$$\int_0^\infty \frac{(x \cos x - \sin x)^2}{x^6} dx = \frac{\pi}{15}.$$
(OR)

- b) i) Show that the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}$ is $e^{\frac{-x^2}{2}}$. (8)
 - ii) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence find the Fourier sine transform of $xe^{-a^2x^2}$. (8)
- 15. a) i) Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z+1)}$ using convolution theorem for Z-transforms. (8)
 - ii) Find the inverse Z-transform of $\frac{z^2-3z}{(z-5)(z+2)}$ using residue theorem. (8)
 - b) i) Solve: $y_{n+2} 4y_{n+1} + 4y_n = 0$, $y_0 = 1$, $y_1 = 0$, using Z-transform. (10)
 - ii) Find the Z-transform of $\{n\}$ and $\left\{\frac{1}{n+1}\right\}$. (6)