

**UNIT – III**  
**THREE DIMENSIONAL GRAPHICS**  
**PART-A**

**1. What are blobby objects?**

Some objects do not maintain a fixed shape, but change their surface characteristics in certain motions or when in proximity with other objects. These objects are referred to as blobby objects, since their shapes show a certain degree of fluidness.

**2. What are spline curves? (AU NOV/DEC 2011 & NOV/DEC 2012)**

The term spline is a flexible strip used to produce a smooth curve through a designated set of points. In computer graphics, the term spline curve refers to any composite curve formed with polynomial sections satisfying specified continuity conditions at the boundary of the pieces.

**3. How to generate a spline curve?**

A spline curve is specified by giving a set of coordinate positions called as control points. These control points are then fitted with piece wise continuous parametric polynomial functions in one of the two ways. When polynomial sections are fitted so that the curve passes through each control point, the resulting curve is said to interpolate the set of control points. When the polynomials are fitted to the general control point path without necessarily passing through any control point the resulting curve is said to approximate the set control points.

**4. What are called control points?**

The spline curve is specified by giving a set of coordinate positions, called control points, which indicate the general shape of the curve.

**5. When is the curve said to interpolate the set of control points?**

When polynomial sections are fitted so that the curve passes through each control point, the resulting curve is said to interpolate the set of control points.

**6. When is the curve said to approximate the set of control points?**

When the polynomials are fitted to the general control-point path without necessarily passing through any control point, the resulting curve is said to approximate the set of control points.

**7. What is called a convex hull?**

The convex polygon boundary that encloses a set of control points is called the convex hull.

**8. Explain about Bezier curves.**

This is a spline approximation method. A Bezier curve section can be fitted to any number of control points. The number of control points to be approximated and their relative position determine the degree of the Bezier polynomial. As with the interpolation splines, a Bezier curve can be specified with boundary conditions, with a characterization matrix, or with blending functions.

**9. What are the various 3D transformations?**

The various 3D transformations are translation, reflection, scaling, rotation and shearing.

**10. What is shear transformation? (AU MAY/JUNE 2012 IT)**

Shearing transformations can be used to modify object shapes. They are also used in 3D viewing for obtaining general projection transformation. A z-axis 3D shear:

$$SH_z = \begin{pmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Parameters a and b can be assigned any real value.

**11. Define viewing. (AU MAY/JUNE 2012)**

Viewing in 3D have more parameters to select when specifying how a 3D scene is to be mapped to a display device. The scene description must be processed through the viewing coordinate transformation and projection routines that transform the 3D viewing coordinate into 2D device coordinates.

**12. Mention some surface detection methods.**

Back-face detection, depth-buffer method, A-buffer method, scan-line method, depth-sorting method, BSP-tree method, area subdivision, octree method, ray casting.

**13. What is ray casting?**

Ray casting methods are commonly used to implement constructive solid geometry operations when objects are described with boundary representations. Ray casting is applied by constructing composite objects in world coordinates with the xy plane corresponding to the pixel plane of a video monitor. This plane is referred to as "firing plane", since each pixel emits a ray through the objects that are combined. Then the surface intersections along each ray path, and they are sorted according to the distance from the firing plane. The surface limits for the composite objects are determined by specified set operations.

**14. What are the two types of projections?**

Parallel projection: coordinate positions are transformed to the view plane along parallel lines.

Perspective projection: object positions are transformed to the view plane along lines that converge to a point called projection reference point.

**15. Differentiate parallel projection from perspective projection. (AU MAY/JUNE 2012)**

Parallel Projection	Perspective Projection
In parallel projection, coordinate positions are transformed to the view plane along parallel lines.	In perspective projection, object positions are transformed to the view plane along lines that converge to a point called projection reference point or center of projection
Preserves the relative proportions of objects.	Produce realistic views but does not preserve relative proportions.
Used in drafting to produce scale drawings of 3D objects.	Projections of distant objects are smaller than the projections of objects of the same size that are closer to the projection plane.

**16. Differentiate oblique and orthographic parallel projections. (AU MAY/JUNE 2012 IT & NOV/DEC 2012)**

Orthographic Parallel Projection	Oblique Parallel projection
Projection is perpendicular to the view plane.	Projection is not perpendicular to the view plane.
Used to produce front, side and top views of object called as elevations.	An oblique projection vector is specified with two angles, $\alpha$ and $\beta$ .

**17. What are the two types of parallel projection?**

Orthographic parallel projection: projection is perpendicular to the view plane.

Oblique parallel projection: projection is not perpendicular to the view plane.

**18. What is axonometric projection?**

Orthogonal projections that display more than one face of an object are axonometric projection.

**19. What is isometric projection?**

Isometric projection is obtained by aligning the projection plane so that it intersects each coordinate axis in which the object is defined at the same distance from the origin.

**20. What are cavalier projections?**

Point  $(x, y, z)$  is projected to position  $(x_p, y_p)$  on the view plane. The projection line from  $(x, y, z)$  and  $(x_p, y_p)$  makes an angle  $\alpha$  with the line on the projection plane that joins  $(x_p, y_p)$  and  $(x, y)$ . When  $\alpha = 45^\circ$  the views obtained are cavalier projections. All lines perpendicular to the projection plane are projected with no change in length.

**21. What are the representation schemes for solid objects?**

Boundary representations: they describe a 3D object as a set of surfaces that separate the object interior from environment. Example: polygon facets  
Space partitioning representations: they are used to describe interior properties, by partitioning the spatial region containing an object into a set of small, non-overlapping, contiguous solids. Example: octree

**22. Define quadric surfaces. (AU NOV/DEC 2011)**

Quadric surfaces are described with second degree equations (quadrics). They include sphere, ellipsoids, tori, paraboloids and hyperboloids. Spheres and ellipsoids are common elements of graphic scenes, they are often available in graphics packages from which more complex objects can be constructed.

**23. What is an ellipsoid?**

An ellipsoid surface can be described as an extension of a spherical surface, where the radii in three mutually perpendicular directions can have different values. The parametric representation for ellipsoid of

latitude angle  $\phi$  and longitude angle  $\theta$  is  $x = r_x \cos\phi \cos\theta$ ,  $-\pi/2 \leq \phi \leq \pi/2$ ,  $y = r_y \cos\phi \sin\theta$ ,  $-\pi \leq \theta \leq \pi$  and  $z = r_z \sin\phi$

**24. Define Octree.**

Hierarchical tree structures called octrees are used to represent solid objects in some graphics system. The tree structure is organized so that each node corresponds to a region of 3D space. This representation for solids takes advantage of spatial coherence to reduce storage requirements for 3D objects.

**25. Write about sweep representations.**

Sweep representations are useful for constructing three-dimensional objects that possess translational, rotational or other symmetries. One can represent such objects by specifying a 2D shape and a sweep that moves the shape through a region of space. A set of 2D primitives, such as circle and rectangles, can be provided for sweep representations as menu options.

**PART - B****1. Differentiate parallel and perspective projections and derive their projection matrices. (AU NOV/DEC 2011 & MAY/JUNE 2012 IT & NOV/DEC 2012)**

- Parallel projections:
  - no shortening due to distance
  - several kinds, depending on orientation:
    - isometric, cavalier,...
- Perspective projections:
  - shortening of objects in the distance
  - several kind, depending on orientation:
    - one, two, three vanishing points

Parallel Projection Matrix

- Parallel projection onto  $z=0$  plane:  $x'=x, y'=y, w'=w$

Matrix for this projection:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective Projection Matrix

Projection onto plane  $z=0$ , with center of projection at  $z=-d$ :

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

Perspective projections pros and cons:

Size varies inversely with distance - looks realistic – Distance and angles are not (in general) preserved – Parallel lines do not (in general) remain parallel

Parallel projection pros and cons:

Less realistic looking + Good for exact measurements + Parallel lines remain parallel – Angles not (in general) preserved

Parallel projections

For parallel projections, we specify a direction of projection (DOP) instead of a COP. There are two types of parallel projections: w Orthographic projection — DOP perpendicular to PP w Oblique projection — DOP not perpendicular to PP There are two especially useful kinds of oblique projections: w Cavalier projection • DOP makes  $45^\circ$  angle with PP • Does not foreshorten lines perpendicular to PP w Cabinet projection • DOP makes  $63.4^\circ$  angle with PP • Foreshortens lines perpendicular to PP by one half

Perspective in the graphic arts is an approximate representation, on a flat surface (such as paper), of an image as it is seen by the eye. The two most characteristic features of perspective are that objects are smaller as their distance from the observer increases; and that they are foreshortened, meaning that an object's dimensions along the line of sight are shorter than its dimensions across the line of sight.

**2. Explain about 3D object representations. (AU MAY/JUNE 2012)**

Polygon surfaces-polygon tables-plane equations-polygon meshes

Object descriptions are stored as sets of surface polygons

The surfaces are described with linear equations

Polygon table

data is placed into the polygon table for processing

Polygon data table can be organised into two groups

geometric table

attribute table

Quadric surfaces-sphere-ellipsoid-torus

Described with second degree eqns.

Ex. Sphere, ellipsoids, tori, paraboloids, hyperboloids

Sphere

A spherical surface with radius 'r' centered on the coordinate origin is defined as a set of points(x,y,z) that satisfy the equation

$$x^2 + y^2 + z^2 = r^2$$

In parametric form,

$$x = r \cos \Phi \cos \Theta$$

$$y = r \cos \Phi \sin \Theta$$

$$z = r \sin \Phi$$

Bloppy objects-definition and example

Don't maintain a fixed shape

Change surface characteristics in certain motions

Ex. Water droplet, Molecular structures

$$f(x,y,z) = \sum_k b_k e^{-a_k r^2} - T = 0$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

T = some threshold

a, b used to adjust the amount of blobiness.

Spline-representation-interpolation

it is a composite curve formed with polynomial pieces satisfying a specified continuity conditions at the boundary of the pieces

Bezier curves

can be fitted to any no. of control points

degree of bezier polynomial is determined by the number of control points and their relative position

Bezier curve is specified by

Boundary conditions

Characterising matrix

Blending function

### 3. How are polygon surfaces represented in 3D?

Polygon tables-Basic concept Polygon table data is placed into the polygon table for processing Polygon data table can be organised into two groups geometric table attribute table Storing geometric data To store geometric data three lists are created

Vertex table – contains coordinate values for each vertex

Edge table – contains pointers back into the vertex table

Polygon table – contains pointers back into the edge table

Advantages of three table

efficient display of objects

For faster info. Extraction

expand edge table to include forward pointers to the polygon table Plane Equation

$$Ax + By + Cz + D = 0$$

eqn. is solved by Cramer's rule

Identification of points

if  $Ax + By + Cz + D < 0$ , the points (x,y,z) is inside the surface

if  $Ax + By + Cz + D > 0$ , the points (x,y,z) is outside the surface

### 4. Write notes on quadric surfaces. (AU NOV/DEC 2012)

Quadric surfaces-definition Described with second degree eqns.

Ex. Sphere, ellipsoids, tori, paraboloids, hyperboloids Sphere-definition-equations-diagram

Sphere

A spherical surface with radius 'r' centered on the coordinate origin is defined as a set of points(x,y,z) that satisfy the equation

$$x^2 + y^2 + z^2 = r^2$$

$$x = r \cos \Phi \cos \Theta$$

$$y = r \cos \Phi \sin \Theta$$

$$z = r \sin \Phi$$

Ellipsoid-definition-equations-diagram

Ellipsoid

Extension of spherical surface ,where the radii in three mutually perpendicular directions have different values

$$(x/r_x)^2 + (y/r_y)^2 + (z/r_z)^2 = 1$$

**5. With suitable examples, explain all 3D transformations. (AU NOV/DEC 2011 & MAY/JUNE 2012 IT & NOV/DEC 2012 & MAY/JUNE 2012)**

Transformation-definition and types Translation-definition-equations-diagram-matrix representation Translation

$$PI = T .P$$

$$xI = x + tx$$

$$yI = y + ty$$

$$zI = z + tz$$

Inverse translation

- obtained by negating translation distances Rotation-definition-equations-diagram-matrix representation Rotation

To perform rotation we need,

An axis

Rotation angle

+ve rotation angles produce counter clockwise rotation

-ve rotation angles produce clockwise rotation

Coordinate axis rotation Z-axis, Y-axis and X-axis

Z axis rotation

$$xI = x \cos \Theta - y \sin \Theta$$

$$yI = x \sin \Theta + y \cos \Theta$$

$$zI = z$$

$$PI = Rz(\Theta).P$$

Scaling Reflection Shearing -definition

Scaling:

alters the size of the object

coordinate values of the vertex is multiplied by scaling factors  $S_x$  &  $S_y$   $xI = x \cdot S_x$

$$yI = y \cdot S_y$$

Reflection

produces mirror image

obtained by rotating the object 180 degrees about the reflection axis.

Shear

distorts the shape of an object.

can be with respect to both axis

Reflection-definition-equations-diagram-matrix representation Shearing-definition-equations-diagram-matrix representation

**6. Write notes on 3D viewing. (AU NOV/DEC 2012)**

Viewing – transfers positions from world coordinate plane to pixels positions in the plane of the output device

Viewing pipeline:

$$MC \rightarrow MT \rightarrow WC \rightarrow VT \rightarrow VC \rightarrow PT \rightarrow PC \rightarrow WT \rightarrow DC$$

Transformation from world to viewing coordinates:

sequences

Translate view reference point to the origin of world coordinate system Apply rotation to align  $x_v$ ,  $y_v$ ,  $z_v$  axes with the world  $x_w$ ,  $y_w$ ,  $z_w$  axes

**7. Discuss the various surface detection methods in detail. (AU MAY/JUNE 2012 IT)**

**Back face detection**

A point  $(x,y,z)$  is inside a polygon surface with plane parameters  $A,B,C$  and  $D$  if

$$Ax+By+Cz+D < 0$$

When an inside point is along the line of sight to the surface, the polygon must be a back-face

Conditions for back face:

A polygon is a back-face if  $V.N > 0$

**Depth buffer method****Steps**

Initialize the depth buffer and refresh buffer so that for all the buffer positions (x,y)

$\text{depth}(x,y) = 0$  ,  $\text{refresh}(x,y) = I \text{ backgnd}$

For each position on each polygon surface listed on the polygon table calculate the depth value and compare the depth value to the previously stored values in the depth buffer to determine visibility

Let the calculated depth be Z for each position (x,y)

If  $Z > \text{depth}(x,y)$  , then set )  $\text{depth}(x,y) = Z$  ,  $\text{refresh}(x,y) = I \text{ surf}(x,y)$

Scan-line method-concept-example-diagram

Extension of scan line algorithm for filling polygon interiors All polygon surfaces intersecting the scan lines are examined

Depth calculations are made for each overlapping surface across every scan line to determine the nearest surface to the view plane

After the visible surface is determined the intensity value for the position is entered into the refresh buffer

**Depth-sorting method****Steps:**

Surfaces are ordered according to the largest Z value

Surface S with greatest depth is compared with other surfaces to determine whether there are any overlaps in depth

If no depth overlap occurs , S is scan converted

This process is repeated for the next surface as long as no overlap occurs

If depth overlaps occurred additional comparisons are used to determine whether reordering of surfaces are needed or not

Ray casting method

- it is a variation of depth buffer method

- process pixels one at a time and calculate depths for all surfaces along the projection path to that pixel

Wireframe method

visible edges are displayed and hidden edges are either eliminated or displayed

differently from the visible edges .Procedures for determining visibility of object edges are referred to as wireframe visibility methods / visible line detection methods / hidden line detection methods

**8. Explain in detail about depth buffer method and A-buffer method for visible surface detection.**

Depth buffer method Steps

1. Initialize the depth buffer and refresh buffer so that for all the buffer positions (x,y)

$\text{depth}(x,y) = 0$  ,  $\text{refresh}(x,y) = I \text{ backgnd}$

2. For each position on each polygon surface listed on the polygon table calculate the depth value and compare the depth value to the previously stored values in the depth buffer to determine visibility

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**9. Explain in detail about B-Spline curves and surfaces.****Control Points**

- A set of points that influence the curve's shape

**Knots**

- Control points that lie on the curve

**Interpolating Splines**

- Curves that pass through the control points (knots)

**Approximating Splines**

Control points merely influence shape

B-splines consist of curve segments whose polynomial coefficients depend on just a few control points

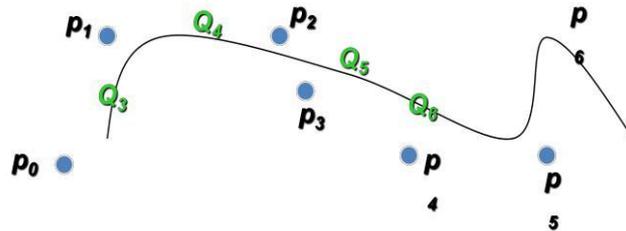
- Local control

Examples of Splines

Start with a sequence of control points

Select four from middle of sequence ( $p_{i-2}$ ,  $p_{i-1}$ ,  $p_i$ ,  $p_{i+1}$ )

- Bézier and Hermite goes between  $p_{i-2}$  and  $p_{i+1}$
- B-Spline doesn't interpolate (touch) any of them but approximates the going through  $p_{i-1}$  and  $p_i$



### Uniform B-Splines

Approximating Splines

Approximates  $n+1$  control points

- $P_0, P_1, \dots, P_n, n, 3$

Curve consists of  $n-2$  cubic polynomial segments

- $Q_3, Q_4, \dots, Q_n$

$t$  varies along B-spline as  $Q_i: t_i \leq t < t_{i+1}$

$t_i$  ( $i = \text{integer}$ ) are knot points that join segment  $Q_{i-1}$  to  $Q_i$

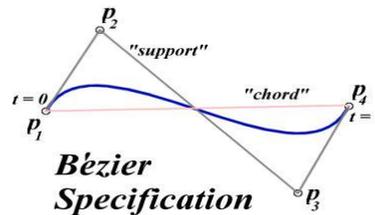
Curve is uniform because knots are spaced at equal intervals of parameter,  $t$  First curve segment,

$Q_3$ , is defined by first four control points

Last curve segment,  $Q_m$ , is defined by last four control points,  $P_{m-3}, P_{m-2}, P_{m-1}, P_m$  Each control point affects four curve segments

### 10. Explain in detail about Bézier curves and surfaces.

Four control points, two of which are knots



The derivative values of the Bézier Curve at the knots are dependent on the adjacent points

$$\nabla_{P_1} = 3(P_2 - P_1)$$

$$\nabla_{P_4} = 3(P_4 - P_3)$$

The scalar 3 was selected just for this curve

- Bézier Blending Functions
- Look at the blending functions
- This family of polynomials is called order-3 Bernstein Polynomials
  - $C(3, k) t^k (1-t)^{3-k}; 0 \leq k \leq 3$
  - They are all positive in interval  $[0, 1]$
  - Their sum is equal to 1