Unit-III :- Random Process.

Random variables, Central limit theorem, Random Process, Stationary processes, Mean, Correlation Covariance functions, Power Spectral Density, Ergodic Processes, Gaussian process, Transmission of a random process through a LTI filter.

Random variables:

Introduction:

*) A function that maps every out comes in a sample space to a real number is called as a random variable of the experiment.

*) Random variables are denoted by the upper case letters such as X, Y etc, and the values taken by them are denoted by lower case letters with subscripts, such as x_1, x_2, y_1, y_2 etc.

*) Random variables may be classified into.

(i) Discrete RV

(ii) continous RV.

Discrete RV:-

*) The random variable which

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$X = \{x_1 \; x_2 \; x_3 \; x_4 \; x_5 \; x_6 \; x_7 \; x_8\}$

$\{3 \; 2 \; 2 \; 2 \; 1 \; 1 \; 1 \; 0\}$

Here the random variable 'X' takes on only a finite number of values, that is, 8. therefore, it is a discrete random variable

(i) Discrete Probability distribution:-

Let x be a discrete random variable and also let $x_1, x_2, x_3 \ldots$ be the values that x can taken, then

$$P(x = x_j) = f(x_j), \quad j = 1, 2, 3 \ldots \quad (1)$$

$f(x_j)$ is the probability of x_j

$f(x_j)$ (or) $f(x)$ — Probability function.

Properties:-

(i) $f(x) \geq 0$ and

(ii) $\sum_x f(x) = 1$

(ii) Cumulative distribution function (CDF):-

The CDF of a random variable 'X' may be defined as the probability that a random variable 'X' takes a values less than (or) equal to x.

$x \rightarrow$ dummy variable.

Now according to the defination, the cumulative distribution function (CDF) may be written as.

$$CDF : F_X(x) = P(X \leq x)$$

Properties:-

(i) $F_X(\infty) = 1$

(ii) $F_X(-\infty) = 0$

(iii) $0 \leq F_X(x) \leq 1$

(iii) Probability density function (PDF):-

The derivative of CDF with respect to some dummy variable 'x' is known as probability density function.

$$PDF : f_X(x) = \frac{d}{dx} F_X(x)$$

Important Random variables:

(1) Bernoulli random variable:

This is discrete random variable taking two values, 1 and 0, with probabilities P and 1-P. It is a good model for a binary data generator.

When binary data is transmitted over a comm channel, some bits are received in error. We can model an error by modulo-2 addition of a 1 to the i/p bit; thus, we change a 0 into 1 and 1 int 0.

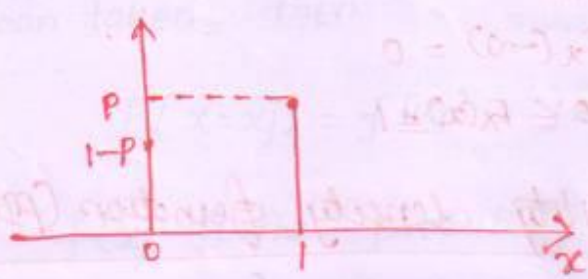

Fig:- PMF for the Bernoulli random variable.

(i) mean = P and

(ii) variable = pq (or) p(1-p).

(ii) Binomial random variable:

Let us consider an experiment with only two possible outcomes, such an experiment is also known as Bernoulli trial.

One outcome is called Success and other is called failure. Let the experiment be repeated a number of times.

$$f(x) = P(X=x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & 0 \le x \le n \\ 0 & ; o.w. \end{cases}$$

(i) mean $= np$

(ii) Variable $= npq$ (or) $np(1-p)$

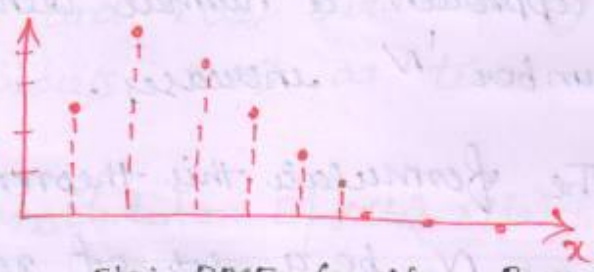

Fig:- PMF for the Binomial random variable.

(iii) Uniform random Variable:

This is a continous random variable taking values B/W a and b with equal probabilities for intervals of equal length.

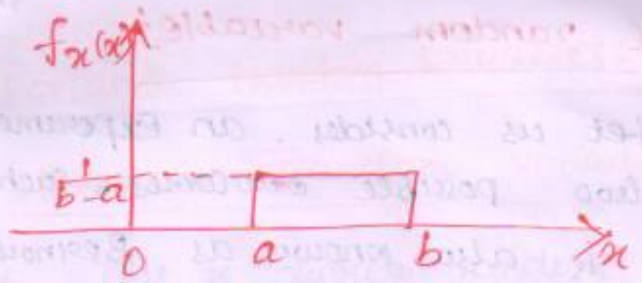

Fig:- PDF for the uniform random variable

Properties:-

(i) mean $= \frac{a+b}{2}$

(ii) variance $= \frac{(b-a)^2}{12}$.

Central limit theorem:

*) The central limit theorem states that the probability density of a sum of N independent random variables tends to approach a normal density as the number 'N' increases.

*) The formulate this theorem, let $x_i = 1, 2, \ldots N$, be a set of random variables that satisfies the following requirements.

(i) The x_i are statistically independent

(ii) The x_i have the same probability

$$Y_i = \frac{1}{\sigma_x}(x_i - \mu_x), \quad i = 1, 2, \ldots N$$

So that we have

$$E[Y_i] = 0$$

and

$$Var[Y_i] = 1$$

Then, the random variable as

$$\boxed{V_N = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} Y_i}$$

Auto Correlation function:

*) The auto correlation function of the process $x(t)$ is the expectation of the product of two random variables $x(t_1)$ and $x(t_2)$ obtained by observing the process $x(t)$ at time t_1 and t_2.

$$R_x(t_1, t_2) = E[x(t_1) \cdot x(t_2)]$$

$$\boxed{R_x(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 \, f_{x(t_1)x(t_2)}(x_1, x_2) \, dx_1 \, dx_2}$$

*) The auto correlation function of a strictly stationary process depends

Auto co-variance function of a strictly sense stationary process $x(t)$ is expressed as.

$$C_x(t_1, t_2) = E\left[(x(t_1) - \mu_x)(x(t_2) - \mu_x)\right]$$

$$\boxed{C_x(t_1, t_2) = R_x(t_2 - t_1) - \mu_x^2}$$

Covariance:

Let μ_x & μ_y be the mean of random variables X & Y, then the central moment of two variables as.

$$E\left[(X-\mu_x)(Y-\mu_y)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (X-\mu_x)(Y-\mu_y) f_{xy}(x, y)\, dx\, dy \quad \text{--- (1)}$$

The above eqn is nothing but covariance of X & Y. It can be expressed as

$$\boxed{\sigma_{x,y} : \operatorname{cov}(x, y) = E\left[(X-\mu_x)(Y-\mu_y)\right]}$$